Figure 1. A taxonomy of methods of representation of cracking in a numerical model.

Figure 2. Databases and processes used during incremental crack growth simulations; ***i*** denotes the increment of crack growth. From Carter, Wawrzynek, and Ingraffea, 2000

Figure 3a. Cross-section of Norfork dam containing initial, vertical crack. From Clough, 1962.

Figure 3b. Finite element mesh of cracked Norfork dam. From Clough, 1962.

Figure 3c. Detail of node decoupling technique used to represent stationary crack in Norfork dam. From Clough, 1962.

Figure 3d. Effect of crack on major principal stress field in Norfork dam. From Clough, 1962.

Figure 4. Example early use of nodal decoupling scheme to model non-colinear crack growth along existing element edges in a finite element model of a reinforced concrete beam. From Ngo and Scordelis (1967).

Figure 5. Prescribed propagation along a symmetry line with a cohesive cracking constitutive model gradually opening interface finite elements.

Figure 6a. Initial FE model for simulation of impact between a steel pellet and an alumina plate. From Camacho and Ortiz, 1996.

Figure 6b. Detail of prescribed growth of multiple cracks along preexisting element edges using a cohesive fracture model. From Camacho and Ortiz, 1996.

Figure 7a. Finite element model for a metallic polycrystal. Interface finite elements have been inserted along all grain boundaries. Ellipse indicates region in which grain boundary decohesion occurred as shown in Figure 7b.

Figure 7b. Prescribed fatigue crack initiation from grain boundaries in a metallic polycrystal.

Figure 8. An example of the analytical geometry version of the constrained shape approach to representation: the finite element alternating method. Case b is the uncracked, structure with finite boundaries, with actual boundary loads and error loads from Case c. Case c is an infinite structure containing the crack in question, loaded with crack-face tractions obtained from Case b. Iterative superposition of Case b and Case c produces the desired situation, Case a. From Wang, Brust, and Atluri, 1997.

Figure 9. Representative structural and crack geometries for which the known solution approach can be used within the NASGRO (1999) code. Left, finite width strip containing a loaded hole. Right, lug with filled, loaded hole.

Figure 10. Representation in element-free, non-colinear crack propagation. Unconnected array of nodal points is adjusted to predicted movement of crack. Layout of near-tip array is important for

accuracy, Figure 11, and a background cell structure is necessary for quadrature, Figure 12. From Belytschko, *et al.*, 1996.

Figure 11. Example use of element-free representation. Left, standard edge crack problem. Right, far- and near-field node arrays and effect of near-field array on accuracy of stress intensity factor calculation. From Belytschko, Lu and Gu, 1994.

Figure 12. Schematic of background cell structure, independent of the nodal arrangement, needed for quadrature in an element-free representation. Note also the special case for domain of quadrature for nodes, x_Q, near a crack. From Belytschko, Lu, Gu, 1994.

Figure 13. Dynamically loaded, single-edge-notch beam simulated with the EFG method in Belytschko, Organ, and Gerlach, 2000. 1 inch = 25.4 mm.

Figure 14. Final crack paths for different initial notch locations predicted with EFG simulation of the beam shown in Figure 12. γ is the distance from the midspan to the starter notch divided by the half supported span. From Belytschko, Organ, and Gerlach, 2000.

Figure 15. 3D crack growth simulation using the EFG coupled with standard FEM. a) Through-crack under combined tension and torsion. b) Overall deflected shape and final crack shape. c) Detail showing final crack shape and local arrangement of EFG nodes and standard FE mesh elements. From Krysl and Belytschko, 1999.

Figure 16. The topology of a through crack, left, and a branching crack, right.

Figure 17. An internal crack can change to a surface crack and, eventually, a complete discontinuity as it propagates.

Figure 18. FE model of a spiral bevel pinion gear. Only 3 teeth modeled, with cracking starting from root of middle tooth, Figure 19.

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Figure 20. Comparison between observed, top, and simulated, bottom, crack trace on surface of middle tooth of gear.

Figure 21. Comparison between observed, top, and simulated, bottom, final fracture surfaces in gear.

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Figure 23. Evolution of mesh model during adaptive FE simulation of crack growth in gear. (a) Detail around initial flaw. (b) Detail at later stage of crack growth.

Figure 24. Segments of triangular lattices of continuous beam elements. (a) a regular lattice. (b) a random lattice. From Chiaia, Vervuurt, and van Mier, 1997.

Figure 25. A lattice model of the brazilian test on a particulate composite. Red indicates particles, yellow interphase area, blue matrix material. (a) before loading. (b) after loading. Images courtesy of J. van Mier.

Figure 26. Local crack tip model using a discrete element approach with circular elements. Red bars indicate inter-element force magnitude and direction. Blue vectors indicate displacement field. From Potyondy and Cundall, 2003.

Figure 27. A bonded particle model of the brazilian test on granite. (a) Overall model after cracking. (b) Detail of central region. Yellow indicates circular particles, white initial voids,

black bars are intact parallel bonds connecting particle centers, red lines (125) are tensile microcracks, blue lines (23) are shear microcracks. From Potyondy and Cundall, 2003.

Figure 28. Discrete, atomistic model of crack growth in {001} single crystal silicon using the Modified Embedded Atom Method. (a) Initial crack length is 10.8 Å, thickness is 20.3 Å. Upper edge atoms displaced at 1 Å/ps. (b) Result after 73 ps. Green atoms have low tensile stresses, blue shades increasingly higher tensile stresses. After Gall, Iesulauro, Hui, and Ingraffea, 2000.

Figure 29. Comparison between generic cohesive fracture approaches for continuum (a), and discrete element approaches (b).

Figure 30. The smeared crack concept. Red lines indicate damage associated with quadrature points, blue. See equation 4 for material stiffness at these points.

Figure 31. A FE model of an asymmetrically loaded beam of concrete. Experimental and FE results using the smeared crack, constitutive approach shown in Figure 32. All dimensions in mm. From Rots, Nauta, and Kusters, 1985.

Figure 32. Central region, FE results using the smeared crack, constitutive approach for the structure shown in Figure 31. (a) crack pattern, left, and deformed shape, right, from baseline mesh. (b) Change in crack pattern and deformed shape resulting from local mesh refinement. (c) Change in crack pattern and deformed mesh resulting from use of inclined mesh. From Rots, Nauta, and Kusters, 1985.

Figure 33. Experimental and FE load versus crack-mouth-sliding-displacement results, using various versions of the smeared crack, constitutive approach for the structure shown in Figure 31. From Rots, Nauta, and Kusters, 1985.

Figure 34. Basis for computational cell approach. (a) a notional sketch of crack growth through void growth and coalescence in a ductile material. (b) Representation of the active layer in (a) with a single layer of computational cells along the crack path. From Ruggieri, Panontin, and Dodds, 1996.

Figure 35. Geometry and FE model for a side-grooved SE(B) specimen used for the computational cell approach. From Ruggieri, Panontin, and Dodds, 1996.

Figure 36. Comparison of measured and predicted R-curves for the side-grooved SE(B) specimen shown in Figure 35. Crack fronts for points A and B shown in Figure 37. From Ruggieri, Panontin, and Dodds, 1996.

Figure 37. Comparison of post-test measured (Joyce and Link, 1995) and predicted crack front profiles for the side-grooved SE(B) specimen shown in Figure 36. (a) Point A in Figure 36, a/W= 0.6 and J= 700 kJ/m2. (b) Point B in Figure 36, a/W= 0.14 and J=1100 kJ/m2, From Ruggieri, Panontin, and Dodds, 1996.

Figure 38. An XFEM crack representation on a structured 2D mesh. Squared nodes are enriched by a displacement jump function, while circled nodes are enriched functions describing the known crack tip displacement field. After Moës, Dolbow and Belytschko, 1999; from Karihaloo and Xiao, 2003a.

Figure 39. A cube containing an inclined, penny-shaped crack subjected to uniaxial tensile load. This is an example problem for the XFEM approach. See Figure 40 for predicted crack shape. From Gravouil, Moës, and Belytschko, 2002.

Figure 40. Evolution of the crack shown in Figure 39 after 17 time steps. From Gravouil, Moës, and Belytschko, 2002.