

Table 1 Normalized stress intensity factors for an edge cracked plate, Figure 23a

L/a	Displacement		MCCI		J -integral	
	Correlation					
	I	% error	I	% error	I	% error
0.5	0.992	-0.81	0.994	-0.65	1.003	0.31
0.25	1.000	0.00	0.994	-0.65	1.002	0.15
0.125	0.999	-0.14	0.994	-0.60	1.001	0.13
0.0625	0.999	-0.09	0.994	-0.65	1.001	0.11

Table 2 Normalized stress intensity factors for a center cracked plate, Figure 23b

L/a	Displacement		MCCI		J -integral	
	Correlation					
	I	% error	I	% error	I	% error
0.5	1.074	7.40	0.986	-1.36	1.005	0.46
0.25	1.038	3.84	0.993	-0.69	1.004	0.43
0.125	1.019	1.87	0.995	-0.47	1.004	0.39
0.0625	1.010	1.02	0.996	-0.43	1.004	0.35

Table 3 Normalized mode I stress intensity factors for a 45° crack in an "infinite" plate.

L/a	Displacement		MCCI		J -integral	
	Correlation					
	I	% error	I	% error	I	% error
0.5	1.066	6.59	0.985	-1.53	1.003	0.27
0.25	1.033	3.30	0.989	-1.11	1.002	0.25
0.125	1.013	1.34	0.992	-0.83	1.002	0.20

Table 4 Normalized mode II stress intensity factors for a 45° crack in an "infinite" plate.

L/a	Displacement		MCCI		J -integral	
	Correlation					
	I	% error	I	% error	I	% error
0.5	0.982	-1.77	0.968	-3.17	1.001	0.14
0.25	0.936	-6.43	0.959	-4.08	0.987	-1.26
0.125	0.905	-9.50	0.948	-5.19	0.971	-2.91

Table 5 Signs for the full expression and the A_4 terms based on the signs of the COD and CSD

		CSD	
		+	-
COD	+	full +	full -
		$A_4 -$	$A_4 -$
	-	full +	full -
		$A_4 +$	$A_4 -$

Table 6 High accuracy solutions for selected test specimens, after Chen *et al.*(2001)

Specimen Description	$K_I / \sigma \sqrt{\pi a}$	T / σ	$B = T \sqrt{\pi a} / K_I$
DCB (h/w=0.2, a/w=0.5)	3.9225	11.5745	2.9508
SENT (h/w=12, a/w=0.3)	1.6598	-0.61033	-0.36771
SENT (h/w=12, a/w=0.5)	2.8246	-0.42168	-0.14929
SENB (h/w=12, a/w=0.3)	1.1241	-0.079177	-0.070436

Table 7 Comparison of computed with exact solutions for K_I and $\delta K_I / \delta a$, Example 1, $p = 1.0$, crack-tip element size= $a / 8$.

a/W	K_I		$\delta K_I / \delta a$	
	MPa√m	MPa√m	MPa√m/m	MPa√m/m
	Exact	Computed (Error %)	Exact	Computed (Error %)
0.010	0.5605	0.5600 (0.09)	2.802	2.759 (1.53)
0.011	0.5879	0.5876 (0.05)	2.672	2.642 (1.12)
0.012	0.6140	0.6134 (0.10)	2.558	2.536 (0.87)
0.013	0.6391	0.6385 (0.09)	2.458	2.409 (1.97)
0.014	0.6632	0.6630 (0.03)	2.369	2.320 (5.05)

Table 8 Comparison of computed with exact solutions for K_I and $\delta K_I / \delta a$, Example 2, $\sigma_0 = 1.0$, crack-tip element size = $a / 8$.

a/W	K_I		$\delta K_I / \delta a$	
	MPa√m	MPa√m	MPa√m/m	MPa√m/m
	Exact	Computed (Error %)	Exact	Computed (Error %)
0.010	0.5605	0.5610 (0.09)	2.802	2.739 (5.25)
0.011	0.5879	0.5884 (0.08)	2.672	2.617 (5.06)
0.012	0.6140	0.6145 (0.08)	2.558	2.561 (1.17)
0.013	0.6391	0.6395 (0.06)	2.458	2.404 (5.20)
0.014	0.6632	0.6627 (0.08)	2.369	2.313 (5.36)

Table 9 Comparison of computed with exact solutions for $\delta^2 K_I / \delta a^2$ for 3 different mesh perturbations, Example 2, $\sigma_0=1.0$, crack-tip element size = $a / 8$.

a/W	Exact	1st ring (Error %)	1st + 2nd rings (Error %)	1st+2nd+3rd (Error %)
0.010	-14.012	-52.373 (273.8)	-17.465 (24.6)	-15.044 (7.37)
0.011	-12.145	-58.599 (382.5)	-15.386 (26.7)	-11.637 (4.18)
0.012	-10.659	-40.801 (282.8)	-13.447 (26.2)	-11.915 (11.8)
0.013	-9.454	-46.023 (386.8)	-12.394 (31.1)	-9.881 (4.52)
0.014	-8.459	-25.607 (167.3)	-10.941 (29.3)	-8.911 (5.34)

Table 10 The computed values of K_I for various crack-tip element sizes in three different mesh perturbation schemes. Example 2, $\sigma_0=1.0$ (Exact value of $K_I = 0.5605$ for $a/W=0.01$).

Element Size	1st ring (Error %)	1st+2nd rings (Error %)	1st+2nd+3rd (Error %)
a/8	0.5625 (0.36)	0.5617 (0.21)	0.5610 (0.09)
a/16	0.5622 (0.30)	0.5613 (0.14)	0.5611 (0.11)
a/32	0.5620 (0.27)	0.5610 (0.09)	0.5607 (0.04)
a/64	0.5617 (0.21)	0.5608 (0.05)	0.5604 (0.02)
a/128	0.5616 (0.20)	0.5606 (0.02)	0.5602 (0.05)

Table 11 The computed values of $\delta K_I / \delta a$ for various crack tip element sizes in three different mesh perturbation schemes. Example 2, $\sigma_0=1.0$ (Exact value of $\delta K_I / \delta a = 2.802$ for $a/W = 0.01$).

Element Size	1st ring	1st+2nd rings	1st +2nd +3rd
	(Error %)	(Error %)	(Error %)
a/8	2.698 (3.71)	2.745 (5.03)	2.739 (5.25)
a/16	2.618 (6.57)	2.744 (5.07)	2.741 (5.18)
a/32	2.501 (10.7)	2.736 (5.36)	2.747 (1.96)
a/64	2.250 (19.7)	2.732 (5.50)	2.747 (1.96)
a/128	1.788 (36.2)	2.732 (5.50)	2.765 (1.32)

Table 12 The computed values of $\delta^2 K_I / \delta a^2$ for various crack tip element sizes in three different mesh perturbation schemes. Example 2, $\sigma_0=1.0$ (Exact value of $\delta^2 K_I / \delta a^2 = -14.013$ for $a/W = 0.01$).

Element Size	1st ring	1st + 2nd rings	1st+2nd+3rd
	(Error %)	(Error %)	(Error %)
a/8	-52.373 (273.8)	-17.465 (24.6)	-15.044 (7.4)
a/16	-67.957 (384.9)	-18.990 (35.5)	-14.967 (6.8)
a/32	-225.25 (1507.4)	-29.614 (111.3)	-14.264 (1.8)
a/64	-856.94 (6015.3)	-79.044 (464.1)	-13.151 (6.2)
a/128	-3404.7 (24196.7)	-281.20 (1906.7)	-17.647 (25.9)

Table 13 Critical states of crack arrest for various temperature profiles, Example 4.

Profile	Results from Bazant (1979a)			Results of present method		
	$a/2h$	$D/2h$	a/D	$a/2h$	$D/2h$	a/D
1	1.53	2.342	0.651	1.53	2.386	0.642
2	3.07	3.907	0.787	3.10	3.973	0.780
3	3.20	3.594	0.891	3.24	3.656	0.886
4	6.97	7.806	0.893	7.03	7.897	0.890

Table 14 The computed values of first order derivatives of stress intensity factors, Example 5 (exact value of $\delta K_I / \delta a = 0.5642$ for axisymmetric case).

Node	j:1	2	3	4	5	6	7	Row sum
i:1	-1.9232	1.4975	0.4725	0.2064	0.1347	0.1040	0.0487	0.5406
2	0.7487	-1.6870	0.8519	0.3036	0.1552	0.1161	0.0520	0.5406
3	0.2362	0.8519	-1.8558	0.8007	0.2850	0.1552	0.0674	0.5406
4	0.1032	0.3036	0.8007	-1.8745	0.8007	0.3036	0.1032	0.5406
5	0.0674	0.1552	0.2850	0.8007	-1.8558	0.8519	0.2362	0.5406
6	0.0520	0.1161	0.1552	0.3036	0.8519	-1.6870	0.7487	0.5406
7	0.0487	0.1040	0.1347	0.2064	0.4725	1.4975	-1.9232	0.5406

Table 15 Stress intensity factors extrapolated to an elliptical crack and compared with reference solution (Irwin, 1962). Example 5 ($b/a=1.2, 1.4$).

θ	$b/a=1.2$			$b/a=1.4$		
	Exact	Predicted	Error %	Exact	Predicted	Error %
0	1.1214	1.1092	-1.09	1.1049	1.0591	-4.14
$\Pi/12$	1.1328	1.1239	-0.78	1.1362	1.1047	-2.77
$\Pi/6$	1.1594	1.1575	-0.16	1.1974	1.1941	-0.28
$\Pi/4$	1.1880	1.1912	0.27	1.2495	1.2624	1.03
$\Pi/3$	1.2104	1.2145	0.34	1.2834	1.2957	0.96
$5\Pi/12$	1.2239	1.2266	0.22	1.3016	1.3073	0.44
$\Pi/2$	1.2284	1.2302	0.15	1.3073	1.3097	0.18

Table 16 Comparison of the calculated values of stress intensity factors with reference solutions (Newman *et al.*, 1981) for semi-circular surface crack. Example 6.

$$(K_0 = 2\sigma\sqrt{a/\pi} \text{ for penny-shaped crack})$$

Node	1	2	3	4	5	6	7
θ	0	$\pi/12$	$\pi/6$	$\pi/4$	$\pi/3$	$5\pi/12$	$\pi/2$
$K_I(\theta)/K_0$	1.1569	1.1171	1.0753	1.0546	1.0434	1.0379	1.0360
K_{REF}/K_0	1.1463	1.0990	1.0675	1.0504	1.0432	1.0414	1.0413
Error (%)	-0.92	-1.65	-0.73	-0.40	-0.03	0.33	0.51

Table 17 The computed rates of stress intensity factors for semi-circular surface crack. Example 6.

Node	j:1	2	3	4	5	6	7	Row sum
i:1	-1.9937	1.5874	0.5424	0.2207	0.1431	0.1107	0.0518	0.6625
2	0.8220	-1.8550	0.9243	0.3473	0.1774	0.1322	0.0593	0.6074
3	0.2918	0.9601	-2.0768	0.8409	0.3171	0.1771	0.0774	0.5876
4	0.1210	0.3679	0.8574	-2.0487	0.8301	0.3317	0.1148	0.5742
5	0.0794	0.1899	0.3268	0.8390	-1.9972	0.8792	0.2526	0.5696
6	0.0617	0.1423	0.1835	0.3370	0.8839	-1.8053	0.7639	0.5669
7	0.0579	0.1278	0.1606	0.2337	0.5086	1.5302	-2.0526	0.5661

Table 18 Comparison of the extrapolated values of stress intensity factors with reference solutions (Newman *et al.*, 1981). Example 6 ($b/a = 1.2, 1.4$).

θ	$b/a=1.2$			$b/a=1.3$		
	Exact	Predicted	Error %	Exact	Predicted	Error %
0	1.3030	1.2981	0.38	1.3015	1.2824	1.47
$\pi/12$	1.2537	1.2530	0.06	1.2584	1.2435	1.18
$\pi/6$	1.2466	1.2459	0.06	1.2698	1.2663	0.28
$\pi/4$	1.2607	1.2606	0.01	1.2992	1.3007	-0.12
$\pi/3$	1.2788	1.2741	0.37	1.3256	1.3215	0.31
$5\pi/12$	1.2919	1.2812	0.83	1.3423	1.3295	0.95
$\pi/2$	1.2966	1.2833	1.03	1.3479	1.3313	1.23

Table 19 The computed values of energy release rates along the crack front for various Poisson's ratios. Example 7 ($W=H=20.0, t=4.0, a=1.0$, 2D reference solution of $G = 3.1681$ for $\nu = 0.0$)

z/t	0.5000	0.4167	0.3333	0.2500	0.1667	0.0833	0.0000
Node	1	2	3	4	5	6	7
$\nu=0.0$	3.1757	3.1757	3.1757	3.1757	3.1757	3.1757	3.1757
$\nu=0.15$	3.2020	3.2128	3.1692	3.1393	3.1226	3.1144	3.1119
$\nu=0.3$	3.1718	3.2308	3.1337	3.0573	3.0106	2.9862	2.9786
$\nu=0.4$	3.0983	3.2253	3.0903	2.9733	2.8991	2.8594	2.8470

Table 20 The computed values of energy release rates along the crack front for various Poisson's ratios. Example 7 ($W = H = 20.0$, $t = 4.0$, $a = 8.0$, 2D reference solution of $G = 37.136$ for $\nu = 0.0$).

z/t	0.5000	0.4167	0.3333	0.2500	0.1667	0.0833	0.0000
Node	1	2	3	4	5	6	7
$\nu=0.0$	37.367	37.367	37.367	37.367	37.367	37.367	37.367
$\nu=0.15$	35.661	36.926	37.418	37.576	37.639	37.665	37.672
$\nu=0.3$	32.738	36.009	37.387	37.865	38.071	38.160	38.186
$\nu=0.4$	29.857	34.992	37.291	38.120	38.490	38.655	38.703

Table 21 The computed values of first order derivatives of energy release rates, Example 7 ($W = H = 20.0$, $t = 4.0$, $a = 1.0$, 2D reference solution of $\delta G / \delta a = 3.2114$ for $\nu = 0.0$)

Node	j:1	2	3	4	5	6	7	Row sum
i:1	-5.9948	5.2121	1.9936	0.8648	0.5150	0.3858	0.1752	3.1518
2	2.6060	-5.8639	3.4901	1.3916	0.7550	0.5320	0.2409	3.1518
3	0.9968	3.4901	-7.1247	3.2745	1.3442	0.8252	0.3458	3.1518
4	0.4324	1.3916	3.2745	-7.247	3.2858	1.4534	0.5610	3.1518
5	0.2575	0.7550	1.3442	3.2858	-7.1528	3.5640	1.0982	3.1518
6	0.1929	0.5320	0.8252	1.4534	3.5640	-6.4154	2.9997	3.1518
7	0.1752	0.4818	0.6915	1.1219	2.1963	5.9993	-7.5143	3.1518

Table 22 Characteristics of FE simulation of crack growth in spiral bevel gear.

Item	Characteristic
Computer power	32 node Pentium III PC-cluster, 733 MHz
Number of unknowns	265,000-395,000
Solution time per load cycle	53-88 min. (212-352 sec/load step)
Typical time for one crack step	~4-6 hours*

*1-1.5 hour solution time plus 3-4.5 hours post-processing and remeshing