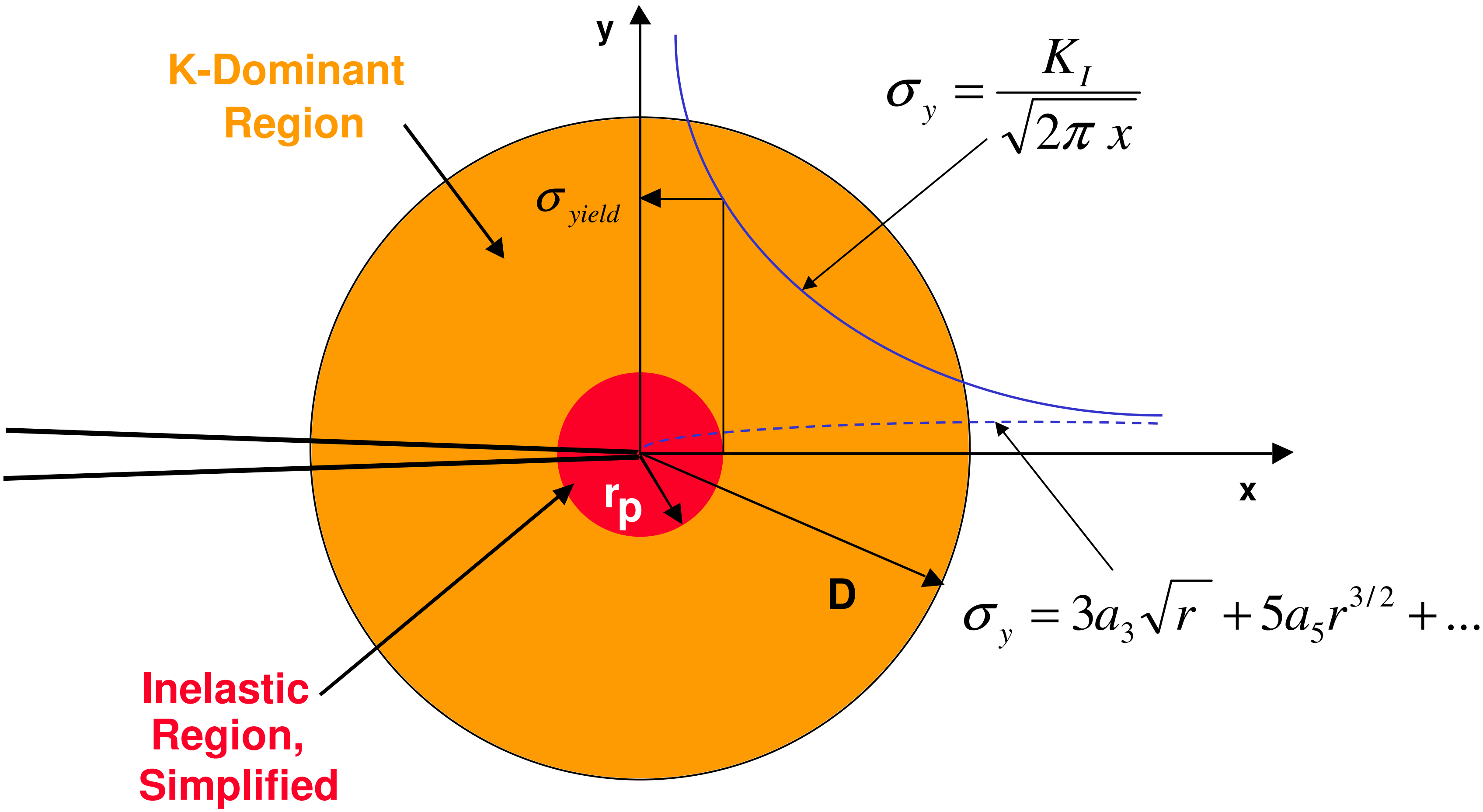
CEE 770 Meeting 5

Objectives of This Meeting

Learn the limit of applicability of LEFM:

- K-dominance
- The process zone, and estimating its size
- Small-scale yielding, SSY
- Dimensional restrictions on process zone size
- The concept of “Brittleness Number”

The Concept of K-Dominance: When is LEFM Applicable?

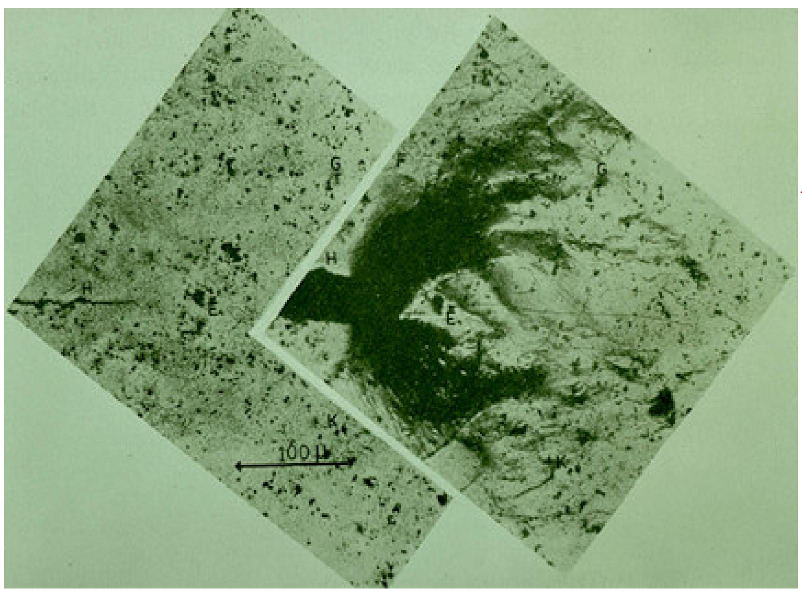


If r_p << D, K_I still controls fracture process.

How big is r_p ????

LEFM predicts infinite stress at the crack front. This cannot be true for a real material.

There must be a region near the crack front where nonlinear material behavior prevails. This behavior might be governed by elasto-plastic, or microcracking, or any other dissipative mechanisms.
This region is called the ***fracture process zone.***



Process zone formation in Al-Cu-Mg alloy, From Broek, Elementary Engineering Fracture Mechanics, 3d Edit, 1982.

Physical Observation

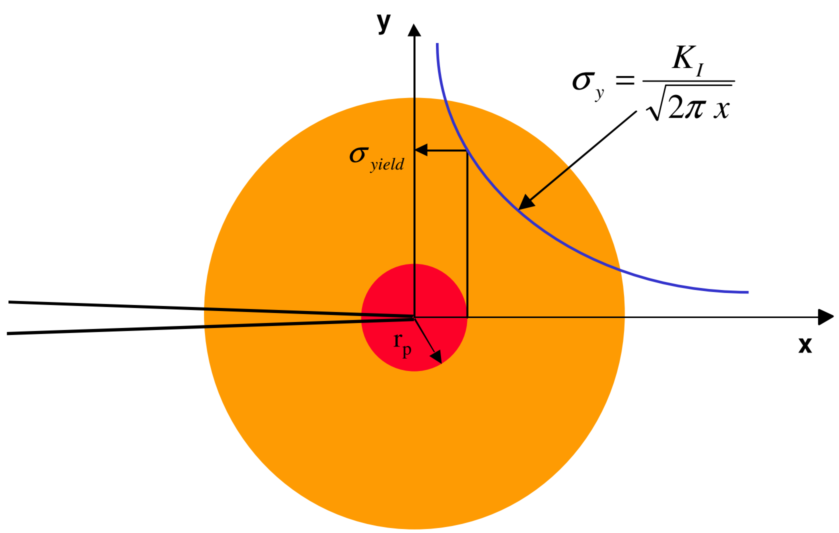
Example of an elasto-plastic process zone

See Computational Simulation

Let's Estimate r_p

First approximation, call it r^*_p:

Ignore triaxiality, assume elastic-perfectly plastic behavior, f_y = uniaxial yield stress, assume no stress redistribution, then



$$\sigma_y = \frac{K_I}{\sqrt{2\pi\, x}}$$

$$\sigma_y = f_y = \frac{K_I}{\sqrt{2\pi\, r_p^*}}$$

$$r_p^* = \frac{K_I^{\,2}}{2\pi\, f_y^{\,2}}$$

And a lower bound on r^*_p

Is this an upper or lower bound to actual value?

$$r_p^* = \frac{K_{Ic}^{\,2}}{2\pi\, f_y^{\,2}} \qquad (62)$$

Better Estimates for r_p

Include 1D stress redistribution:

$$r_p^{**} = \frac{K_{Ic}^{\ 2}}{\pi\, f_y^{\ 2}}$$

Include stress redistribution and allow for multiaxiality with the von Mises yield criterion:

$$r_p = \frac{K_{Ic}^{\ 2}}{3\pi\, f_y^{\ 2}} \qquad \text{in plane strain}$$

Why difference?

$$r_p = \frac{K_{Ic}^{\ 2}}{\pi\, f_y^{\ 2}} \qquad \text{in plane stress}$$

Next, we want r_p to be “small”. Compared to what? How small?

Small Compared to What? How Small?

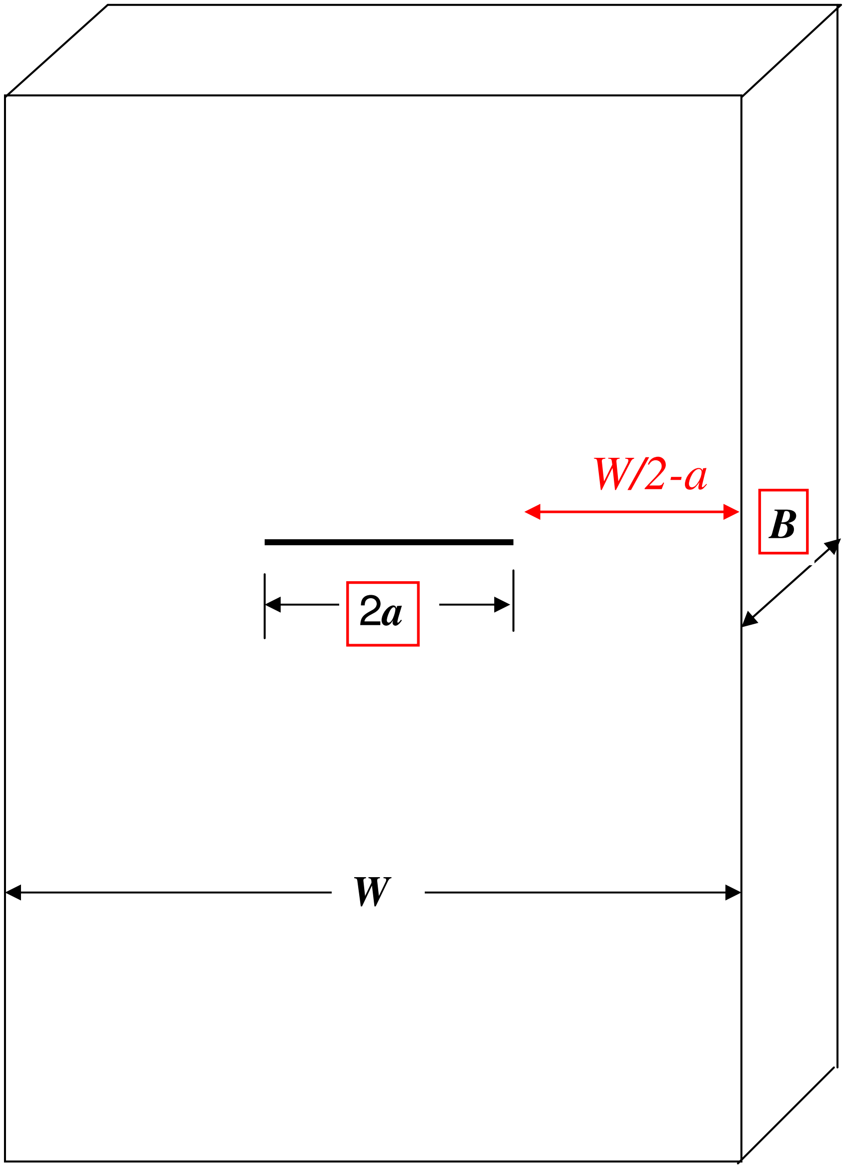
For K, a linear-elastic concept, to dominate the solution, the process zone must be small compared to all significant dimensions of the structure containing the crack.

By convention and by reasonableness, "small" is accepted to mean less than about 1/25th.

For example, we would want r_p to be less than 1/25th of the crack length, a

$$a \geq \frac{25 K_{Ic}^{2}}{3\pi \, f_y^{2}} \quad \text{or} \quad a \geq 2.5\left(\frac{K_{Ic}}{f_y}\right)^2 \tag{63}$$

Necessary Geometrical Conditions for Validity of LEFM in Materials in Which Elasto-Plastic Process Zone Behavior Dominates



Necessary Conditions:

$$a, B, W - a > 2.5\left(\frac{K_{Ic}}{f_Y}\right)^2$$

where

K_{Ic}= Plane Strain Fracture Toughness

f_y= Uniaxial yield strength

W-a = ligament

If all these conditions are met, then SSY conditions are said to prevail and LEFM applies.

Brittleness Numbers

An important concept that emerges from this simple calculation is that of the "Brittleness Number".

A material in a state with a high brittleness number has a small process zone, e.g.

$$r_p \propto \left(\frac{K_{Ic}}{f_y} \right)^2 \qquad \text{for elasto - plastic material state}$$

so the ratio of yield stress to plane strain fracture toughness, f_y/K_{Ic}, can be interpreted as a brittleness number for materials in which the process zone is predominately dissipating energy in plastic strain energy.

Similarly, brittleness numbers for materials which dissipate fracture energy through other mechanisms have been defined, e.g.

$$L_{ch} = EG_F / f_t^2$$

f_t = Tensile strength

G_F = Fracture energy

for concrete which dissipates energy through microcracking.